

**SYSTEM AND METHOD FOR GENERATING A VISUALIZED DATA  
REPRESENTATION PRESERVING INDEPENDENT VARIABLE  
GEOMETRIC RELATIONSHIPS**

**Field of the Invention**

The present invention relates in general to data visualization and, in particular, to a system and method for generating a visualized data representation preserving independent variable geometric relationships.

**Background of the Invention**

Computer-based data visualization involves the generation and presentation of idealized data on a physical output device, such as a cathode ray tube (CRT), liquid crystal diode (LCD) display, printer and the like. Computer systems visualize data through the use of graphical user interfaces (GUIs) which allow intuitive user interaction and high quality presentation of synthesized information.

The importance of effective data visualization has grown in step with advances in computational resources. Faster processors and larger memory sizes have enabled the application of complex visualization techniques to operate in multi-dimensional concept space. As well, the interconnectivity provided by networks, including intranetworks and internetworks, such as the Internet, enable the communication of large volumes of information to a wide-ranging audience. Effective data visualization techniques are needed to interpret information and model content interpretation.

The use of a visualization language can enhance the effectiveness of data visualization by communicating words, images and shapes as a single, integrated unit. Visualization languages help bridge the gap between the natural perception of a physical environment and the artificial modeling of information within the constraints of a computer system. As raw information cannot always be digested

as written words, data visualization attempts to complement and, in some instances, supplant the written word for a more intuitive visual presentation drawing on natural cognitive skills.

Effective data visualization is constrained by the physical limits of computer display systems. Two-dimensional and three-dimensional information can be readily displayed. However,  $n$ -dimensional information in excess of three dimensions must be artificially compressed. Careful use of color, shape and temporal attributes can simulate multiple dimensions, but comprehension and usability become difficult as additional layers of modeling are artificially grafted into the finite bounds of display capabilities.

Thus, mapping multi-dimensional information into a two- or three-dimensional space presents a problem. Physical displays are practically limited to three dimensions. Compressing multi-dimensional information into three dimensions can mislead, for instance, the viewer through an erroneous interpretation of spatial relationships between individual display objects. Other factors further complicate the interpretation and perception of visualized data, based on the Gestalt principles of proximity, similarity, closed region, connectedness, good continuation, and closure, such as described in R.E. Horn, "Visual Language: Global Communication for the 21<sup>st</sup> Century," Ch. 3, MacroVU Press (1998), the disclosure of which is incorporated by reference.

In particular, the misperception of visualized data can cause a misinterpretation of, for instance, dependent variables as independent and independent variables as dependent. This type of problem occurs, for example, when visualizing clustered data, which presents discrete groupings of data which are misperceived as being overlaid or overlapping due to the spatial limitations of a three-dimensional space.

Consider, for example, a group of clusters, each cluster visualized in the form of a circle defining a center and a fixed radius. Each cluster is located some distance from a common origin along a vector measured at a fixed angle from a common axis through the common origin. The radii and distances are independent variables relative to the other clusters and the radius is an

independent variable relative to the common origin. In this example, each cluster represents a grouping of points corresponding to objects sharing a common set of traits. The radius of the cluster reflects the relative number of objects contained in the grouping. Clusters located along the same vector are similar in theme as  
5 are those clusters located on vectors having a small cosine rotation from each other. Thus, the angle relative to a common axis' distance from a common origin is an independent variable with a correlation between the distance and angle reflecting relative similarity of theme. Each radius is an independent variable representative of volume. When displayed, the overlaying or overlapping of  
10 clusters could mislead the viewer into perceiving data dependencies where there are none.

Therefore, there is a need for an approach to presenting arbitrarily dimensioned data in a finite-dimensioned display space while preserving independent data relationships. Preferably, such an approach would maintain size  
15 and placement relationships relative to a common identified reference point.

There is a further need for an approach to reorienting data clusters to properly visualize independent and dependent variables while preserving cluster radii and relative angles from a common axis drawn through a common origin.

### **Summary of the Invention**

20 The present invention provides a system and method for reorienting a data representation containing clusters while preserving independent variable geometric relationships. Each cluster is located along a vector defined at an angle  $\theta$  from a common axis  $x$ . Each cluster has a radius  $r$ . The distance (magnitude) of the center  $c_i$  of each cluster from a common origin and the radius  $r$  are  
25 independent variables relative to other clusters and the radius  $r$  of each cluster is an independent variable relative to the common origin. The clusters are selected in order of relative distance from the common origin and optionally checked for an overlap of bounding regions. Clusters having no overlapping regions are skipped. If the pair-wise span  $s_{ij}$  between the centers  $c_i$  and  $c_j$  of the clusters is  
30 less than the sum of the radii  $r_i$  and  $r_j$ , a new distance  $d_i$  for the cluster is determined by setting the pair-wise span  $s_{ij}$  equal to the sum of the radii  $r_i$  and  $r_j$

and solving the resulting quadratic equation for distance  $d_i$ . The operations are repeated for each pairing of clusters.

5 An embodiment of the present invention is a system and method for providing a perspective-corrected representation of a multi-dimensional cluster rendering. A span between centers for a pair of clusters is measured. Each center has an independent radius and is located in concept space at an independent distance along a vector drawn from a common origin and formed at a dependent angle from a common axis drawn through the common origin. A perspective-corrected independent distance from the common origin for one such cluster is  
10 determined if the span does not substantially equal the sum of the independent radii of the clusters. The perspective-corrected independent distance substantially equals a root of a quadratic equation of the vectors formed by the independent distances of the clusters and angle formed therebetween.

A further embodiment is a system and method for generating a visualized  
15 data representation preserving independent variable geometric relationships. A pair of convex clusters is selected. Each convex cluster is rendered on a display. Each convex cluster has a center of mass located at an original fixed distance from a common origin, and is oriented along a vector formed at a fixed angle from a common polar axis. A span is measured between the centers of mass of  
20 each convex cluster. For each convex cluster, a segment is measured from the center of mass of each convex shape to a point closest to the other convex shape along the span. A new fixed distance from the common origin for the center of mass for one of the convex clusters located along the vector for that convex cluster is evaluated if the span is less than the sum of the segments of the convex  
25 clusters. The pair of convex clusters is displayed rendered using at least the new fixed distance for the center of mass of the one convex cluster.

A further embodiment is a system and method for providing a perspective-corrected representation of a multi-dimensional convex shape rendering. A plurality of shapes is rendered. Each shape defines a convex volume representing  
30 a data grouping located within a multi-dimensional concept space and includes a center of mass logically located within the convex volume. A span is measured

between the centers of mass for a pair of the convex shapes. Each convex shape is located at an independent distance along a vector drawn from a common origin and is formed at an independent angle from a common axis drawn through the common origin. A perspective-corrected independent distance from the common origin for one such convex shape is determined if the span does not substantially equal the sum of the distances of center of mass to point closest to the other convex shape of each convex shape. The perspective-corrected independent distance substantially equals a root of a quadratic equation formed by the independent distances of the convex shapes and angle formed there between.

Still other embodiments of the present invention will become readily apparent to those skilled in the art from the following detailed description, wherein is described embodiments of the invention by way of illustrating the best mode contemplated for carrying out the invention. As will be realized, the invention is capable of other and different embodiments and its several details are capable of modifications in various obvious respects, all without departing from the spirit and the scope of the present invention. Accordingly, the drawings and detailed description are to be regarded as illustrative in nature and not as restrictive.

#### **Brief Description of the Drawings**

FIGURE 1 is a block diagram showing a system for generating a visualized data representation preserving independent variable geometric relationships, in accordance with the present invention.

FIGURE 2 is a data representation diagram showing, by way of example, a view of overlapping clusters generated by the cluster display system of FIGURE 1.

FIGURE 3 is a graph showing, by way of example, the polar coordinates of the overlapping clusters of FIGURE 2.

FIGURE 4 is a data representation diagram showing, by way of example, the pair-wise spans between the centers of the clusters of FIGURE 2.

FIGURE 5 is a data representation diagram showing, by way of example, an exploded view of the clusters of FIGURE 2.



14, including a CRT, LCD display, and the like. As well, a printer (not shown) could function as an alternate display device. The cluster display system 11 includes a processor, memory and persistent storage, such as provided by a storage device 16, within which are stored clusters 17 representing visualized multi-dimensional data. The cluster display system 11 can be interconnected to other computer systems, including clients and servers, over a network 15, such as an intranetwork or internetwork, including the Internet, or various combinations and topologies thereof.

Each cluster 17 represents a grouping of one or more points in a virtualized concept space, as further described below beginning with reference to FIGURE 2. Preferably, the clusters 17 are stored as structured data sorted into an ordered list in ascending (preferred) or descending order. In the described embodiment, each cluster represents individual concepts and themes categorized based on, for example, Euclidean distances calculated between each pair of concepts and themes and defined within a pre-specified range of variance, such as described in common-assigned U.S. Patent Application Serial No.

\_\_\_\_\_, entitled "System And Method For Efficiently Generating Cluster Groupings In A Multi-Dimensional Concept Space," filed August 31, 2001, pending, the disclosure of which is incorporated by reference.

The cluster display system 11 includes four modules: sort 18, reorient 19, display and visualize 20, and, optionally, overlap check 21. The sort module 18 sorts a raw list of clusters 17 into either ascending (preferred) or descending order based on the relative distance of the center of each cluster from a common origin. The reorient module 19, as further described below with reference to FIGURE 10, reorients the data representation display of the clusters 17 to preserve the orientation of independent variable relationships. The reorient module 19 logically includes a comparison submodule for measuring and comparing pairwise spans between the radii of clusters 17, a distance determining submodule for calculating a perspective-corrected distance from a common origin for select clusters 17, and a coefficient submodule taking a ratio of perspective-corrected distances to original distances. The display and visualize module 20 performs the

actual display of the clusters 17 via the display 14 responsive to commands from the input devices, including keyboard 12 and pointing device 13. Finally, the overlap check module 21, as further described below with reference to FIGURE 12, is optional and, as a further embodiment, provides an optimization whereby clusters 17 having overlapping bounding regions are skipped and not reoriented.

The individual computer systems, including cluster display system 11, are general purpose, programmed digital computing devices consisting of a central processing unit (CPU), random access memory (RAM), non-volatile secondary storage, such as a hard drive or CD ROM drive, network interfaces, and peripheral devices, including user interfacing means, such as a keyboard and display. Program code, including software programs, and data are loaded into the RAM for execution and processing by the CPU and results are generated for display, output, transmittal, or storage.

Each module is a computer program, procedure or module written as source code in a conventional programming language, such as the C++ programming language, and is presented for execution by the CPU as object or byte code, as is known in the art. The various implementations of the source code and object and byte codes can be held on a computer-readable storage medium or embodied on a transmission medium in a carrier wave. The cluster display system 11 operates in accordance with a sequence of process steps, as further described below with reference to FIGURE 9.

FIGURE 2 is a data representation diagram 30 showing, by way of example, a view 31 of overlapping clusters 33-36 generated by the cluster display system 11 of FIGURE 1. Each cluster 33-36 has a center  $c$  37-40 and radius  $r$  41-44, respectively, and is oriented around a common origin 32. The center  $c$  of each cluster 33-36 is located at a fixed distance (magnitude)  $d$  45-48 from the common origin 32. Cluster 34 overlays cluster 33 and clusters 33, 35 and 36 overlap.

Each cluster 33-36 represents multi-dimensional data modeled in a three-dimensional display space. The data could be visualized data for a virtual semantic concept space, including semantic content extracted from a collection of documents represented by weighted clusters of concepts, such as described in



commonly-assigned U.S. Patent application Serial No. \_\_\_\_\_,  
entitled "System And Method For Dynamically Evaluating Latent Concepts In  
Unstructured Documents," filed August 31, 2001, pending, the disclosure of  
which is incorporated by reference.

5       FIGURE 3 is a graph 50 showing, by way of example, the polar  
coordinates of the overlapping clusters 33-36 of FIGURE 2. Each cluster 33-36 is  
oriented at a fixed angle  $\theta$  52-55 along a common axis  $x$  51 drawn through the  
common origin 32. The angles  $\theta$  52-55 and radii  $r$  41-44 (shown in FIGURE 2)  
of each cluster 33-36, respectively, are independent variables. The distances  $d$   
10 56-59 represent dependent variables.

Referring back to FIGURE 2, the radius  $r$  41-44 (shown in FIGURE 2) of  
each cluster 33-36 signifies the number of documents attracted to the cluster. The  
distance  $d$  56-59 increases as the similarity of concepts represented by each  
cluster 33-36 decreases. However, based on appearance alone, a viewer can be  
15 misled into interpreting cluster 34 as being dependent on cluster 33 due to the  
overlay of data representations. Similarly, a viewer could be misled to interpret  
dependent relationships between clusters 33, 35 and 36 due to the overlap  
between these clusters.

FIGURE 4 is a data representation diagram 50 showing, by way of  
20 example, the pair-wise spans between the centers of the clusters of FIGURE 2.  
Centers  $c$  37-40 of the clusters 33-36 are separated respectively by pair-wise  
spans  $s$  60-65. Each span  $s$  60-65 is also dependent on the independent variables  
radii  $r$  41-44 (shown in FIGURE 2) and angles  $\theta$  52-55.

For each cluster 33-36, the radii  $r$  is an independent variable. The  
25 distances  $d$  56-59 and angles  $\theta$  52-55 are also independent variables. However,  
the distances  $d$  56-59 and angles  $\theta$  52-55 are correlated, but there is no correlation  
between different distances  $d$  56-59. As well, the relative angles  $\theta$  52-55 are  
correlated relative to the common axis  $x$ , but are not correlated relative to other  
angles  $\theta$  52-55. However, the distances  $d$  56-59 cause the clusters 33-36 to  
30 appear to either overlay or overlap and these visual artifacts erroneously imply  
dependencies between the neighboring clusters based on distances  $d$  56-59.

FIGURE 5 is a data representation diagram 70 showing, by way of example, an exploded view 71 of the clusters 33-36 of FIGURE 2. To preserve the relationships between the dependent variables distance  $d$  and span  $s$ , the individual distances  $d$  56-59 (shown in FIGURE 3) are multiplied by a fixed coefficient to provide a proportionate extension  $e$  71-75, respectively, to each of the distances  $d$  56-59. The resulting data visualization view 71 “explodes” clusters 33-36 while preserving the independent relationships of the radii  $r$  41-44 (shown in FIGURE 2) and angles  $\theta$  52-55 (shown in FIGURE 3).

Although the “exploded” data visualization view 71 preserves the relative pair-wise spans  $s$  60-65 between each of the clusters 33-36, multiplying each distance  $d$  56-59 by the same coefficient can result in a potentially distributed data representation requiring a large display space.

FIGURE 6 is a data representation diagram 80 showing, by way of example, a minimized view 81 of the clusters 33-36 of FIGURE 2. As in the exploded view 71 (shown in FIGURE 4), the radii  $r$  41-44 and angles  $\theta$  52-55 of each cluster 33-36 are preserved as independent variables. The distances  $d$  56-59 are independent variables, but are adjusted to correct to visualization. The “minimized” data representation view 81 multiplies distances  $d$  45 and 48 (shown in FIGURE 2) by a variable coefficient  $k$ . Distances  $d$  46 and 47 remain unchanged, as the clusters 34 and 35, respectively, need not be reoriented. Accordingly, the distances  $d$  45 and 48 are increased by extensions  $e'$  82 and 83, respectively, to new distances  $d'$ .

FIGURE 7 is a graph 90 showing, by way of example, the polar coordinates of the minimized clusters 33-36 of FIGURE 5. Although the clusters 33-36 have been shifted to distances  $d'$  106-109 from the common origin 32, the radii  $r$  41-44 and angles  $\theta$  102-105 relative to the shared axis  $x$  101 (shown in FIGURE 2) are preserved. The new distances  $d'$  106-109 also approximate the proportionate pair-wise spans  $s'$  110-115 between the centers  $c$  37-40.

FIGURE 8 is a data representation diagram 110 showing, by way of example, the pair-wise spans between the centers of the clusters of FIGURE 2. Centers  $c$  37-40 (shown in FIGURE 2) of the clusters 33-36 are separated

respectively by pair-wise spans  $s$  111-116. Each span  $s$  111-116 is dependent on the independent variables radii  $r$  41-44 and the angles  $\theta$  52-55 (shown in FIGURE 3). The length of each pair-wise span  $s$  111-116 is proportionately increased relative to the increase in distance  $d$  56-69 of the centers  $c$  37-40 of the clusters 33-36 from the origin 32.

FIGURE 9 is a flow diagram showing a method 120 for generating a visualized data representation preserving independent variable geometric relationships, in accordance with the present invention. As a preliminary step, the origin 32 (shown in FIGURE 2) and  $x$ -axis 51 (shown in FIGURE 3) are selected (block 121). Although described herein with reference to polar coordinates, any other coordinate system could also be used, including Cartesian, Logarithmic, and others, as would be recognized by one skilled in the art.

Next, the clusters 17 (shown in FIGURE 1) are sorted in order of relative distance  $d$  from the origin 32 (block 122). Preferably, the clusters 17 are ordered in ascending order, although descending order could also be used. The clusters 17 are reoriented (block 123), as further described below with reference to FIGURE 10. Finally, the reoriented clusters 17 are displayed (block 124), after which the routine terminates.

FIGURE 10 is a flow diagram showing a routine 130 for reorienting clusters 17 for use in the method 120 of FIGURE 9. The purpose of this routine is to generate a minimized data representation, such as described above with reference to FIGURE 5, preserving the orientation of the independent variables for radii  $r$  and angles  $\theta$  relative to a common  $x$ -axis.

Initially, a coefficient  $k$  is set to equal 1 (block 131). During cluster reorientation, the relative distances  $d$  of the centers  $c$  of each cluster 17 from the origin 32 is multiplied by the coefficient  $k$ . The clusters 17 are then processed in a pair of iterative loops as follows. During each iteration of an outer processing loop (blocks 132-146), beginning with the innermost cluster, each cluster 17, except for the first cluster, is selected and processed. During each iteration of the inner processing loop (blocks 135-145), each remaining cluster 17 is selected and reoriented, if necessary.

Thus, during the outer iterative loop (blocks 132-146), an initial *Cluster<sub>i</sub>* is selected (block 133) and the radius  $r_i$ , center  $c_i$ , angle  $\theta_i$ , and distance  $d_i$  for the selected *Cluster<sub>i</sub>* are obtained (block 134). Next, during the inner iterative loop (blocks 135-145), another *Cluster<sub>j</sub>* (block 136) is selected and the radius  $r_j$ , center  $c_j$ , angle  $\theta_j$ , and distance  $d_j$  are obtained (block 137).

In a further embodiment, bounding regions are determined for *Cluster<sub>i</sub>* and *Cluster<sub>j</sub>* and the bounding regions are checked for overlap (block 138), as further described below with reference to FIGURE 14.

Next, the distance  $d_i$  of the cluster being compared, *Cluster<sub>i</sub>*, is multiplied by the coefficient  $k$  (block 139) to establish an initial new distance  $d'_i$  for *Cluster<sub>i</sub>*. A new center  $c_i$  is determined (block 140). The span  $s_{ij}$  between the two clusters, *Cluster<sub>i</sub>* and *Cluster<sub>j</sub>*, is set to equal the absolute distance between center  $c_i$  plus center  $c_j$ . If the pair-wise span  $s_{ij}$  is less than the sum of radius  $r_i$  and radius  $r_j$  for *Cluster<sub>i</sub>* and *Cluster<sub>j</sub>*, respectively (block 143), a new distance  $d_i$  for *Cluster<sub>i</sub>* is calculated (block 144), as further described below with reference to FIGURE 11. Processing of each additional *Cluster<sub>i</sub>* continues (block 145) until all additional clusters have been processed (blocks 135-145). Similarly, processing of each *Cluster<sub>j</sub>* (block 146) continues until all clusters have been processed (blocks 132-146), after which the routine returns.

FIGURE 11 is a flow diagram showing a routine 170 for calculating a new distance for use in the routine 130 of FIGURE 10. The purpose of this routine is to determine a new distance  $d'_i$  for the center  $c_i$  of a selected *cluster<sub>i</sub>* from a common origin. In the described embodiment, the new distance  $d'_i$  is determined by solving the quadratic equation formed by the distances  $d_i$  and  $d_j$  and adjacent angle.

Thus, the sum of the radii  $(r_i + r_j)^2$  is set to equal the square of the distance  $d_j$  plus the square of the distance  $d_i$  minus the product of the 2 times the distance  $d_j$  times the distance  $d_i$  times  $\cos \theta$  (block 171), as expressed by equation (1):

$$(r_i + r_j)^2 = d_i^2 + d_j^2 - 2 \cdot d_i d_j \cos \theta \quad (1)$$

The distance  $d_i$  can be calculated by solving a quadratic equation (5) (block 172), derived from equation (1) as follows:

$$1 \cdot d_i^2 + (2 \cdot d_j \cos \theta) \cdot d_i = (d_j^2 - [r_i + r_j]^2) \quad (2)$$

$$1 \cdot d_i^2 + (2 \cdot d_j \cos \theta) \cdot d_i - (d_j^2 - [r_i + r_j]^2) = 0 \quad (3)$$

$$d_i = \frac{(2 \cdot d_j \cos \theta) \pm \sqrt{(2 \cdot d_j \cos \theta)^2 - 4 \cdot 1 \cdot (d_j^2 - [r_i + r_j]^2)}}{2 \cdot 1} \quad (4)$$

$$d_i = \frac{(2 \cdot d_j \cos \theta) \pm \sqrt{(2 \cdot d_j \cos \theta)^2 - 4 \cdot (d_j^2 - [r_i + r_j]^2)}}{2} \quad (5)$$

- 5 In the described embodiment, the ‘ $\pm$ ’ operation is simplified to a ‘+’ operation, as the distance  $d_i$  is always increased.

Finally, the coefficient  $k$ , used for determining the relative distances  $d$  from the centers  $c$  of each cluster 17 (block 139 in FIGURE 10), is determined by taking the product of the new distance  $d_i$  divided by the old distance  $d_i$  (block 10 173), as expressed by equation (6):

$$k = \frac{d_{i_{new}}}{d_{i_{old}}} \quad (6)$$

The routine then returns.

In a further embodiment, the coefficient  $k$  is set to equal 1 if there is no overlap between any clusters, as expressed by equation (7):

$$15 \quad \text{if } \frac{d_{i-1} + r_{i-1}}{d_i - r_i} > 1, \text{ then } k = 1 \quad (7)$$

where  $d_i$  and  $d_{i-1}$  are the distances from the common origin and  $r_i$  and  $r_{i-1}$  are the radii of clusters  $i$  and  $i-1$ , respectively. If the ratio of the sum of the distance plus the radius of the further cluster  $i-1$  over the difference of the distance less the radius of the closer cluster  $i$  is greater than 1, the two clusters do not overlap and 20 the distance  $d_i$  of the further cluster need not be adjusted.

FIGURE 12 is a graph showing, by way of example, a pair of clusters 181-182 with overlapping bounding regions generated by the cluster display system 11 of FIGURE 1. The pair of clusters 181-182 are respectively located at distances  $d$  183-184 from a common origin 180. A bounding region 187 for cluster 181 is 25 formed by taking a pair of tangent vectors 185a-b from the common origin 180. Similarly, a bounding region 188 for cluster 182 is formed by taking a pair of

tangent vectors 186a-b from the common origin 180. The intersection 189 of the bounding regions 187-188 indicates that the clusters 181-182 might either overlap or overlay and reorientation may be required.

FIGURE 13 is a graph showing, by way of example, a pair of clusters 191-192 with non-overlapping bounding regions generated by the cluster display system 11 of FIGURE 1. The pair of clusters 191-192 are respectively located at distances  $d$  193-194 from a common origin 190. A bounding region 197 for cluster 191 is formed by taking a pair of tangent vectors 195a-b from the common origin 190. Similarly, a bounding region 198 for cluster 192 is formed by taking a pair of tangent vectors 196a-b from the common origin 190. As the bounding regions 197-198 do not intersect, the clusters 191-192 are non-overlapping and non-overlaid and therefore need not be reoriented.

FIGURE 14 is a flow diagram showing a routine 200 for checking for overlap of bounding regions for use in the routine 130 of FIGURE 10. As described herein, the terms *overlap* and *overlay* are simply referred to as "overlapping." The purpose of this routine is to identify clusters 17 (shown in FIGURE 1) that need not be reoriented due to the non-overlap of their respective bounding regions. The routine 200 is implemented as an overlap submodule in the reorient module 19 (shown in FIGURE 1).

Thus, the bounding region of a first  $Cluster_i$  is determined (block 201) and the bounding region of a second  $Cluster_j$  is determined (block 202). If the respective bounding regions do not overlap (block 203), the second  $Cluster_j$  is skipped (block 204) and not reoriented. The routine then returns.

FIGURE 15 is a data representation diagram 210 showing, by way of example, a view 211 of overlapping non-circular cluster 213-216 generated by the clustered display system 11 of FIGURE 1. Each cluster 213-216 has a center of mass  $c_m$  217-220 and is oriented around a common origin 212. The center of mass as  $c_m$  of each cluster 213-216 is located at a fixed distance  $d$  221-224 from the common origin 212. Cluster 218 overlays cluster 213 and clusters 213, 215 and 216 overlap.

As described above, with reference to FIGURE 2, each cluster 213-216 represents multi-dimensional data modeled in a three-dimension display space. Furthermore, each of the clusters 213-216 is non-circular and defines a convex volume representing a data grouping located within the multi-dimensional concept space. The center of mass  $c_m$  at 217-220 for each cluster 213-216, is logically located within the convex volume. The segment measured between the point closest to each other cluster along a span drawn between each pair of clusters is calculable by dimensional geometric equations, as would be recognized by one skilled in the art. By way of example, the clusters 213-216 represent non-circular shapes that are convex and respectively comprise a square, triangle, octagon, and oval, although any other form of convex shape could also be used either singly or in combination therewith, as would be recognized by one skilled in the art.

Where each cluster 213-216 is not in the shape of a circle, a segment is measured in lieu of the radius. Each segment is measured from the center of mass 217-220 to a point along a span drawn between the centers of mass for each pair of clusters 213-216. The point is the point closest to each other cluster along the edge of each cluster. Each cluster 213-216 is reoriented along the vector such that the edges of each cluster 213- 216 do not overlap.

While the invention has been particularly shown and described as referenced to the embodiments thereof, those skilled in the art will understand that the foregoing and other changes in form and detail may be made therein without departing from the spirit and scope of the invention.